Optimization Design for SRM based on the Regulation Model of Ampere Density and Coil Space Factor

Peng Sun, Chi Zhang*, *Senior Member, IEEE,* Jinhua Chen, Zhe Jiang

Ningbo Institute of Materials Technology& Engineering. Chinese Academy of Sciences, CO 315201,China
Zhejiang Key Laboratory of Robotics and Intelligent Manufacturing Equipment Technology, CO 315201, China
E-mail: zhangchi@nimte.ac.cn

Abstract — In the preliminary design process of switched reluctance machine (SRM), adjusting the number of wingding turns or cross-sectional area of conductor to optimize windings design has dramatic effects on the performances like torque density, efficiency and thermal dissipation and so on. However, the difficulty exists on how to guarantee these performances while optimizing winding design. This paper proposes an analytical optimization design method based on an regulation model of ampere density and coil space factor, which can directly determine the optimal number of winding turns, the cross-sectional area of conductor, the ampere density and the coil space factor. And a MATLAB pre-design program has been developed to provide two design schemes respectively with and without the optimization model. The comparison analysis has been further carried out with finite element analysis (FEA). The simulation results verify that the performances can be highly improved with the proposed optimization model.

***Keywords*—Switched reluctance machine, optimization design, ampere density, coil space factor Introduction**

I. Introduction

Switched reluctance machine (SRM) has the concentrated windings on the stator but no permanent magnets and windings on the rotor. It is an attractive alternative for electric vehicle (EV) and hybrid electric vehicle (HEV) applications owing to its simple and low cost construction, wide constant power range, low inertia and low inrush current et al[1].

In many applications, it demands for high torque density, high efficiency, high output power, excellent thermal dissipation and low cost, which can be improved through comprehensive optimization design of SRM and its optimal control. Because the excitation and unique inductance profile is closely determined by the rotor position, the mathematical model of SRM is highly nonlinear and the flux linkage is easy to saturated at the poles, which makes it harder to design and adjust the geometric dimensions to meet the design specifications. Some works focus on the comprehensive design procedure and program for common SRM design, which allows an efficient and fast design by understanding the influence of relevant design parameters. Berker Bilgin et al. have proposed a MATLAB based design tool to calculate the geometry with the conventional magnetic circuit method and automatically draws and simulates the SRM model in FEA software without considering the optimization design[2]. And some works focus on the geometry and configuration optimization to improve the performances like efficiency, torque density and power output et al. Kazuhiro Ohyama et al. have studied the approach to improve the efficiency through designing the cross sections and axial shapes of rotor and stator cores using the magnetic field analysis with 2D and 3D finite element methods[3]. J. Hur et al. have implemented the flux barriers to improve the torque density[4]. Anas Labak et al. have proposed a design and analysis method of a novel axial-flux SRM, in which the detailed procedures of deriving the output power equation as a function of the motor dimensions and parameters and an exclusive pole-shape design are presented. It focuses on the structural design to improve the performance[5]. Dong-Hee Lee et al. have proposed the first and second optimization process to reduce the torque ripple and improve the output torque through optimizing the air gap which is considered to be a function of the rotor position[6].

However, the empirical values are generally needed to guide and amend the design process which always needs abundant design experiences. One of the crucial design procedure is to design and optimize the winding, i.e., the number of winding turns, slot fill factor, ampere density and wire diameter. And winding design and optimization also has great effects on the SRM performances[7-9]. Therefore, this paper mainly focuses on the winding design and optimization issue, and proposes a optimization regulation model based on the ampere density and coil space factor optimization, which is actually a winding design and optimization approach, aiming at improving the torque density and the output power as well as the efficiency through choosing optimal number of winding turns, wire diameter, ampere density and coil space factor. An comparison study with magnetic circuit method and FEM simulation also has been carried out. It verifies that the proposed optimization model is not only advantageous for improvement of torque density, but also for power output and efficiency.

II. Optimization Regulation Model

Ampere density and coil space factor are two crucial factors that must be comprehensively investigated and designed in certain appropriate intervals. It means that , on the other hand, the ampere density and coil space factor are non-unique. There are various combinations of these two factors to

*Corresponding author: Chi Zhang is with the Ningbo Institute of Materials Technology & Engineering, Chinese Academy of Sciences and Zhejiang Key Laboratory of Robotics and Intelligent Manufacturing Equipment Technology, Zhejiang 315201 China (Research Professor , email: zhangchi@nimte.ac.cn). Chi Zhang and Peng Sun contributed equally.

Peng Sun is with the Ningbo Institute of Materials Technology & Engineering, Chinese Academy of Sciences and Zhejiang Key Laboratory of Robotics and Intelligent Manufacturing Equipment Technology Zhejiang 315201 China (PhD student, email: sunpeng@nimte.ac.cn)

meet the same design specifications and requirements. Therefore, guessing that it exists an optimal point, the problem is simply how to determine it. However, actually, whether there is an optimal point or not, it depends on the optimizable range, which is further determined by the preliminary structural design of SRM. The ampere density and coil space factor can be expressed as

$$J = \frac{I}{S_a} = \frac{\pi D_a A}{q} \frac{1}{N_{ph} S_a} \tag{1}$$

$$k_s = \frac{S_{cu}}{S_W} = \frac{1}{2S_W} N_{ph} S_a \tag{2}$$

where J is the ampere density, k_s is the coil space factor, N_{ph} is the windings turns in series of each phase, D_a is the rotor outside diameter, A is the current per unit length, q is the number of phase, I is the effective winding current, S_w is the stator yoke window area, S_{cu} is the net conductor cross-sectional area of each slot and S_a is the cross area of conductor. Corresponding definitions are illustrated in Figure 1. And the relationship between the ampere density and coil space factor is shown as Figure 2.

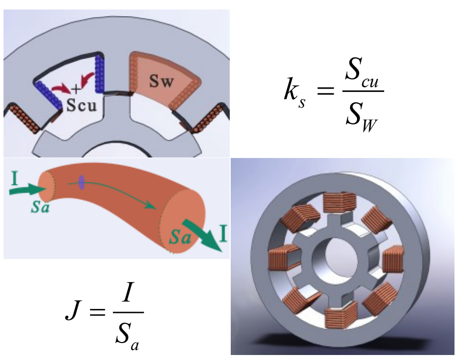


Figure 1 Definitions of ampere density and coil space factor

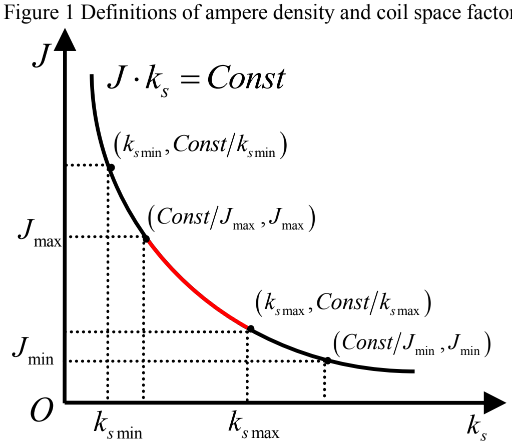


Figure 2 Relationship between ampere density and coil space factor

From (1) and (2), it is obvious that the ampere density and coil space factor can be regulated by changing the number of winding turns and cross-sectional area of conductor assuming that D_a, A, q and S_w change little and can approximately be considered to be constant. It hence yields that

$$J \cdot k_s = \frac{\pi D_a A}{2qS_W} = \text{Const} \tag{3}$$

Only when the intervals of J and k_s having overlapping section on the curve, there is an optimal point. And the overlapping intervals is the so called optimizable range. More detail discussion of the optimizable range can be found in reference[10]. Once the optimization possibility has been discussed, another problem is how to find the optimal point. So, an optimized regulation model has been proposed, which adapts the electromagnetic torque T_{em} as the optimization objective function. According to the co-energy theory[11], the electromagnetic torque can be calculated with co-energy W'.

$$T_{em} = \frac{qN_r}{2\pi} W' \tag{4}$$

where N_r is the number of rotor poles. Figures 3 shows the magnetization curves at aligned position θ_a and unaligned position θ_u, in which the shaded area represents the co-energy, where I_m^* is the ideal square wave peak current, i_s is the critical saturation current for $\psi_a(\theta_a, i)$, L_u is the inductance at θ_u position, L_{ao} is the inductance at θ_a position when $i < i_s$.

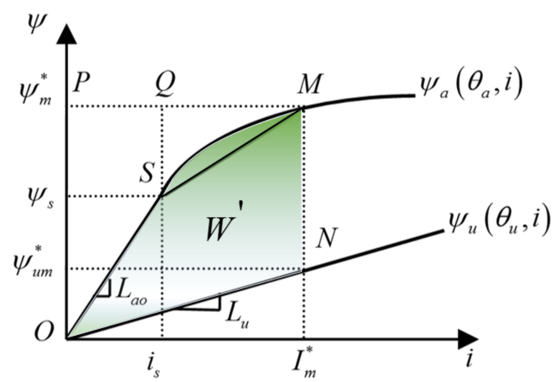


Figure 3 Magnetization curves at aligned (θ_a) and unaligned (θ_u) positions.

Shown as Figure 3, the co-energy can be expressed as

$$\begin{aligned} W' &= \int_0^{I_m^*} \left[\psi_a(\theta_a, i) - \psi_u(\theta_u, i) \right] \mathrm{d}i \\ &\approx S_{OPMN} - S_{OPQS} - S_{SQM} \\ &= \frac{1}{2}\psi_m^* \left(I_m^* - i_s \right) - \frac{1}{2}\psi_{um}^* I_m^* + \frac{1}{2}\psi_s I_m^* \end{aligned} \tag{5}$$

On the other hand, the co-energy can also be expressed as a function of the number of winding turns in series of each phase [10, 12],

$$W' = \begin{cases} P_1 \dfrac{1}{N_{ph}} + P_2 \dfrac{1}{N_{ph}^3} + P_3 & \text{when } \theta_{on} \in [\theta_0, \theta_u] \\ P_4 \dfrac{1}{N_{ph}} + P_5 & \text{when } \theta_{on} \in [\theta_u, \theta_1] \end{cases} \tag{6}$$

where P_1~P_5 are constants which are related to the flux and inductance at special rotor positions[10]. Thus, ultimately, the electromagnetic torque T_{em} can be expressed in the form of N_{ph} as follows,

$$T_{em}=\begin{cases}\dfrac{qN_r}{2\pi}\left(P_1\dfrac{1}{N_{ph}}+P_2\dfrac{1}{{N_{ph}}^3}+P_3\right) & \text{when } \theta_{on}\in\left[\theta_0,\theta_u\right]\\ \dfrac{qN_r}{2\pi}\left(P_4\dfrac{1}{N_{ph}}+P_5\right) & \text{when } \theta_{on}\in\left[\theta_u,\theta_1\right]\end{cases} \tag{7}$$

The practical task of the model is to determine the optimal ampere density J and coil space factor k_s by simply regulating S_a and N_{ph} in the optimization range, meanwhile guaranteeing providing optimal electromagnetic torque. Therefore, the optimized regulation model can be described as

$$\begin{cases} find\ \left(N_{ph},S_a\right)\\ \left(T_{em}\right)_{max}=\max\left[T_{em}\left(N_{ph},S_a\right)\right]\\ J_{min}\le J\left(N_{ph},S_a\right)\le J_{max}\\ k_{s\,min}\le k_s\left(N_{ph},S_a\right)\le k_{s\,max}\end{cases} \tag{8}$$

There are also some constraint conditions as follows,

$$\left(T_{em}\right)_{max}>T_{emN} \tag{9}$$

$$\left(S_aN_{ph}\right)_{min}\le S_aN_{ph}\le\left(S_aN_{ph}\right)_{max} \tag{10}$$

$$J=\frac{I}{S_a}=\frac{I_m^*/\sqrt{2}}{S_a} \tag{11}$$

Thus, the optimization regulation model can ultimately be expressed as a standard nonlinear optimization model shown as

$$\begin{cases}\left(T_{em}\right)_{max}=\max\left[T_{em}\left(N_{ph}\right)\right]\\ J\cdot k_s=\pi D_a A/2qS_W\\ J=I_m^*/\sqrt{2}S_a\\ \left(T_{em}\right)_{max}>T_{emN}\\ \left(S_aN_{ph}\right)_{min}\le S_aN_{ph}\le\left(S_aN_{ph}\right)_{max}\\ J_{min}\le J\le J_{max}\\ k_{s\,min}\le k_s\le k_{s\,max}\end{cases} \tag{12}$$

This is a standard nonlinear optimization model, which can be solved with various numerical computing methods like the Kuhn-Tucker (K-T) equation method and the Sequential Quadratic Programming (SQP) methods. There are also corresponding commands in MATLAB.

III. OPTIMIZATION DESIGN PROGRAM WITH MATLAB

In order to verify the feasibility of the optimization regulation model, a pre-design program has been completed with MATLAB, which utilizes the conventional design approach and procedure to determine the initial geometric dimensions[13, 14]. It provides two preliminary designs, one of which is with the optimization regulation model, and the other is without the model. Figure 4 shows the program flow chart with the optimization regulation model. And the one with no optimization is the same with the procedure shown as in the dashed box.

Corresponding preliminary design specifications and parameters without the optimization regulation model are listed in TABLE I. Compared with the design without the optimization regulation model, the designed structural parameters with the model are the same excepting the winding and electromagnetic parameters as shown in TABLE II.

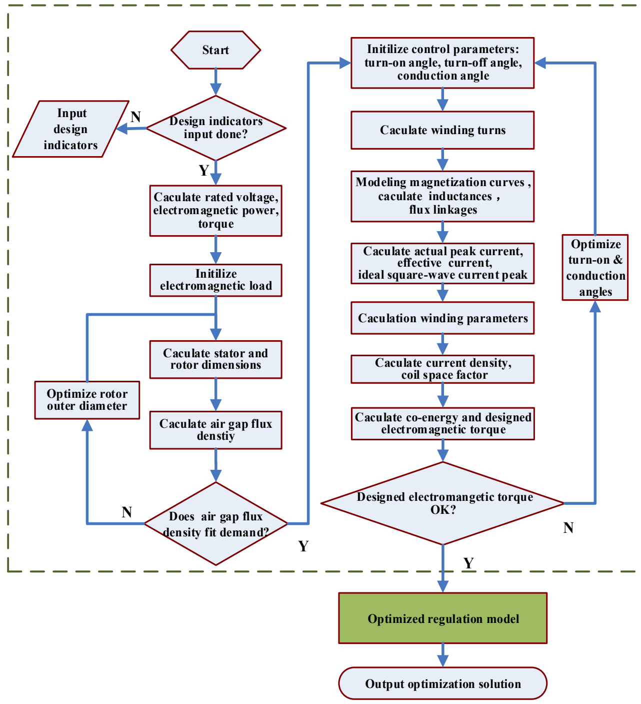


Figure 4 MATLAB program flow chart

TABLE I Design specifications and parameters

Design Specifications			
Number of phases	4	Number of stator and rotor poles	8/6
Rated voltage	380VAC	Rated speed	1500rpm
Rated output power	11KW	Rated efficiency	88%
Design Parameters			
Turn-on angle	-2.35deg	Turn-off angle	18.5deg
Rotor inner diameter	59.5mm	Stator outer diameter	210mm
Rotor yoke height	16.60mm	Core lamination length	178.5mm
Stator pole width	21.83mm	Stator pole arc	21deg
Rotor pole width	23.72mm	Rotor pole arc	23deg
Air gap length	0.4mm	Rotor outer diameter	119mm
Stator slot depth	30.19mm	Stator yoke height	14.19mm
Stator pole pitch	62.31mm		

TABLE II Theoretical performances comparison

Effects of the optimization regulation model		
Nominal	Before Optimized	After Optimized
Winding turns in series of each phase	124	116
Cross area of conductor(mm2)	3.9408	4.9087
Wire diameter(mm)	2.24	2.5
Ampere density(A/mm2)	5.53	4.79
Coil space factor	0.5277	0.6051
Rated electromagnetic torque(N.m)	75.02	99.19

The optimizable range is shown in Figure 5. The coil space factor k_s and the ampere density J are expected to be designed in an certain appropriate interval, which is $k_s \in [0.5, 0.7]$ and $J \in [3.5, 5.5]$. After optimized, the coil space factor is k_s=0.6051, and the ampere density is J=4.79A/mm^2. It means that both k_s and J are in the optimizable range. And Figure 6 shows the theoretical relationship between the electromagnetic torque and the number of winding turns. It states that the number of winding turns in series of each phase N_{ph} should be in the interval of $N_{ph} \in [116, 156]$ when the cross-sectional area of conductor is S_a=4.9087mm^2, and the corresponding electromagnetic torque theoretically varies from 75.02N.m to 100.5N.m as shown in Figure 6. It means that the theoretical value of optimal electromagnetic torque should between 75.02N.m and 100.5N.m, and TABLE II shows that the theoretical optimal electromagnetic torque is 99.19N.m, which is in the optimizable range.

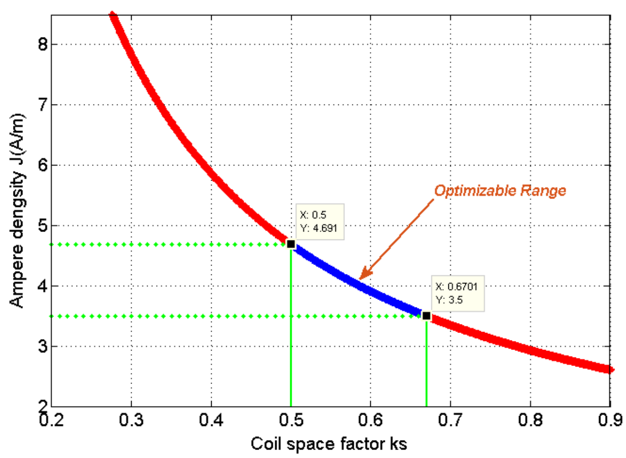


Figure 5 Ampere density vs. coil space factor (Optimizable range)

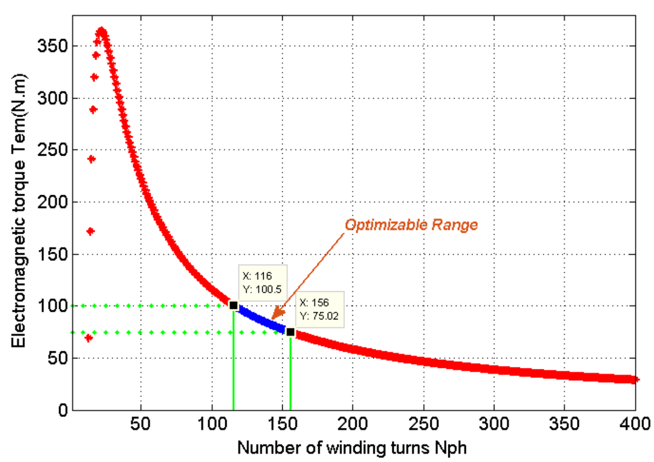


Figure 6 Electromagnetic torque vs. the number of winding turns

It states that the optimization regulation model functions well. It can effectively search an theoretical optimal point in the optimizable range. Before optimized, the designed rated electromagnetic torque is theoretically about 75.02N.m, while after optimized, it increases to 99.19N.m, which can theoretically improved by 32.22%.

IV. Comparison Analysis

And a performances comparison analysis has been carried out with ANSYS Maxwell. The purpose is merely comparatively study the effects of the optimization regulation model on the performances of SRM. The winding configuration here is the conventional type of long magnetic circuit loop shown as Figure 7， of which the corresponding excitation sequence A-A&B-B-B&C-C-C&D-D-D&A.

Figure 8 shows the finite element mesh result, in which the maximum elements length of the coils is 9.02mm, and 9.1mm of stator and rotor. The maximum surface deviation is 0.105mm, and the deviation angle is 15deg.

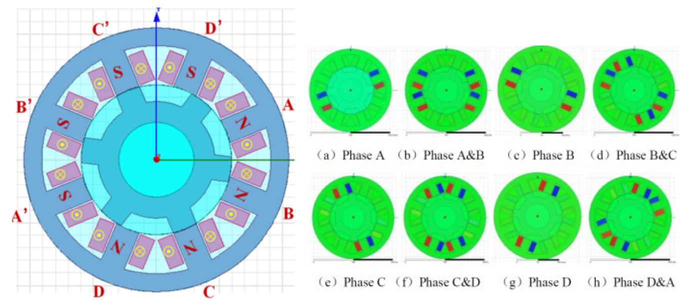


Figure 7 Winding configuration and excitation phase sequence

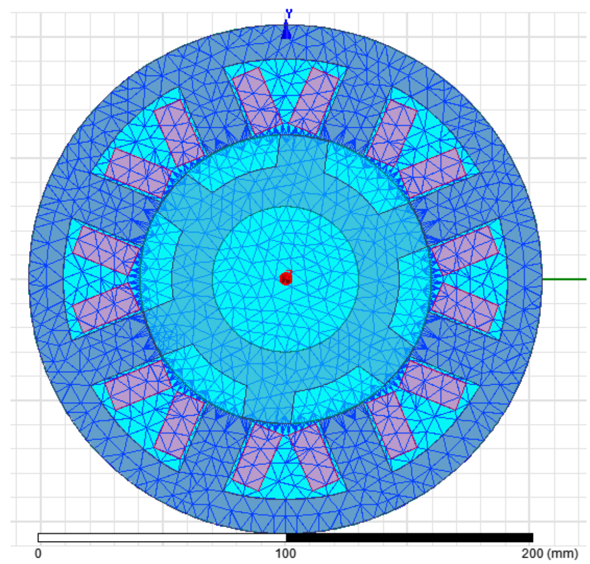
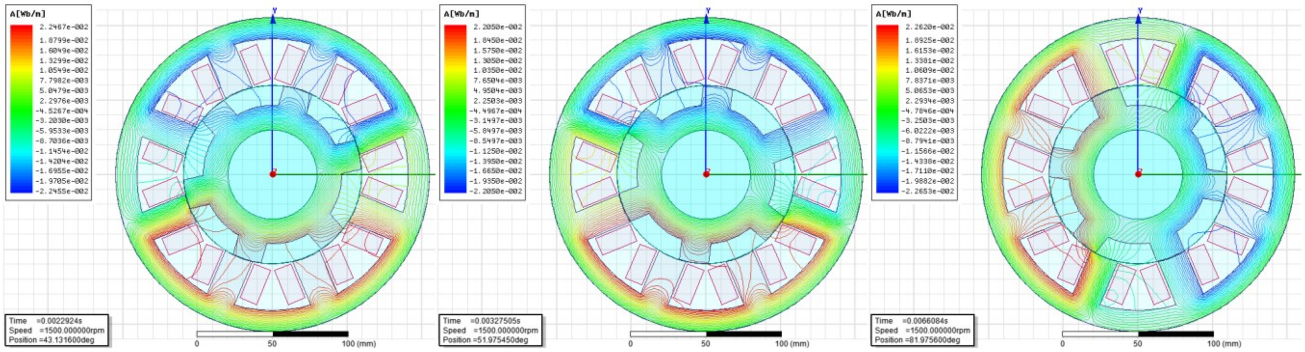


Figure 8 Finite element mesh

Figure 9(a)~(c) and Figure 10(a)~(c) respectively show the flux linkage and density distribution at certain special rotor positions including the partial overlapped, aligned and unaligned positions with respect to the pole of phase-A. Specifically, when the stator pole and the rotor overlapping, the flux density at corresponding overlapped part is relatively larger than that at aligned or unaligned positions. And TABLE III lists the average flux linkage and flux density comparison.

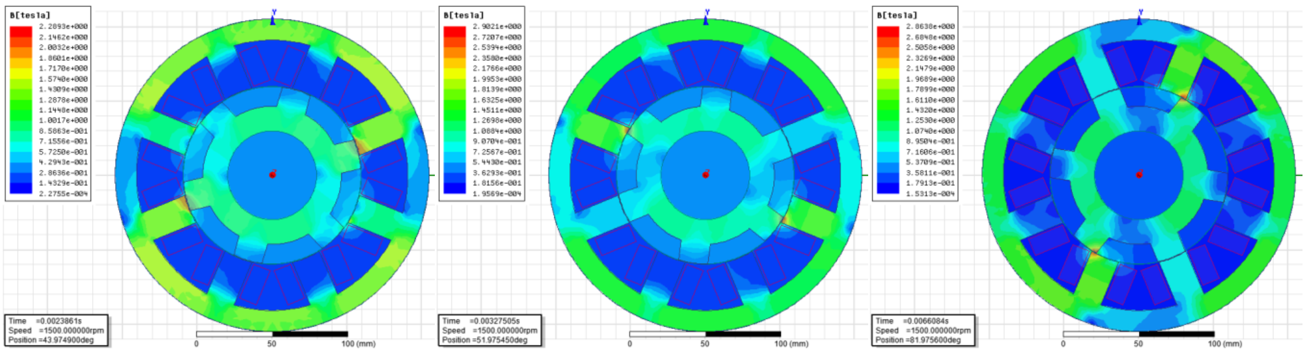
TABLE III Flux linkage and flux density comparison

Nominal	Before Optimized	After Optimized
Flux linkage(Wb)	0.682204	0.640847
Stator-pole flux density(T)	1.64944	1.65630
Stator-yoke flux density(T)	1.14319	1.14795
Rotor-pole flux density(T)	1.63482	1.64162
Rotor-yoke flux density(T)	0.977221	0.981288



(a) partial overlapped- θ_{hr} (phase A&B conducted) (b) aligned- θ_a (phase B conducted) (c) unaligned- θ_a (phase D conducted)

Figure 9 Flux linkage distribution(1500rpm)



(a) partial overlapped- θ_{hr} (phase A&B conducted) (b) aligned- θ_a (phase B conducted) (c) unaligned- θ_a (phase D conducted)

Figure 10 Flux density distribution(1500rpm)

TABLE IV lists the theoretical performances comparison. It shows that the performances has indeed improved after optimized.

TABLE IV Simulation performances comparison

Performances comparison		
Nominal	Before optimized	After optimized
Rated speed(rpm)	1761	1884
Rated output torque(N.m)	69.35	69.97
Output torque at 1500rpm(N.m)	91.87	103.06
Estimated start torque(N.m)	540.85	669.45
Rated electromagnetic power(KW)	13.86	14.84
Rated output power(KW)	12.79	13.81
Maximum output power(KW)	16.91	18.41
Winding copper loss(W)	688.10	597.01
Total loss(W)	1074.55	1026.65
Efficiency(%)	92.25	93.08

Figure 11 shows the simulation output torque curve at different speed, which states that the output torque has been highly improved. When applied the same rated load toque of 70N.m, it is obvious that after optimized, the steady speed increases from 1761rpm to 1884rpm, which means that the operation range has been improved with the optimization regulation model. Correspondingly, the rated output mechanical torque increased from 69.35N.m to 69.97N.m. Besides, before optimized, the output torque is 91.87N.m at 1500rpm, while after optimized, it increases to 103.06N.m, which is improved by 12.18%.

The simulation output power also has been improved shown as Figure 12. The output power at 1500rpm is about 14.34KW before optimized and 16.09KW after optimized, which has been increased by 12.2%.

Shown as Figure 13, it states that under the speed of 1500rpm, the efficiency is not significantly improved, while above the rated speed it obviously increases compared with the circumstance before optimized.

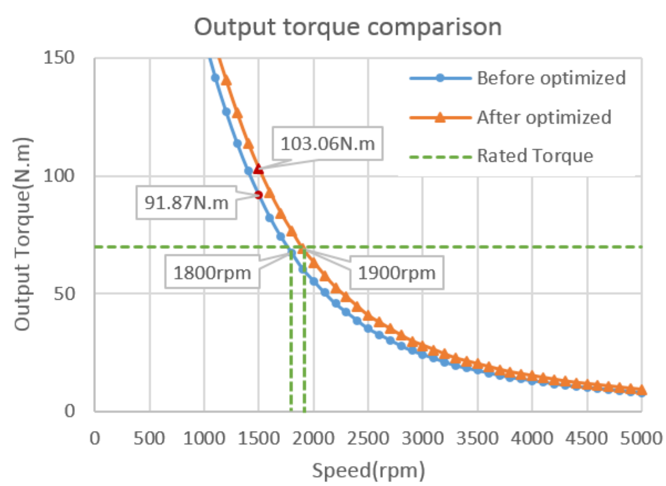


Figure 11 Output torque comparison with various speed

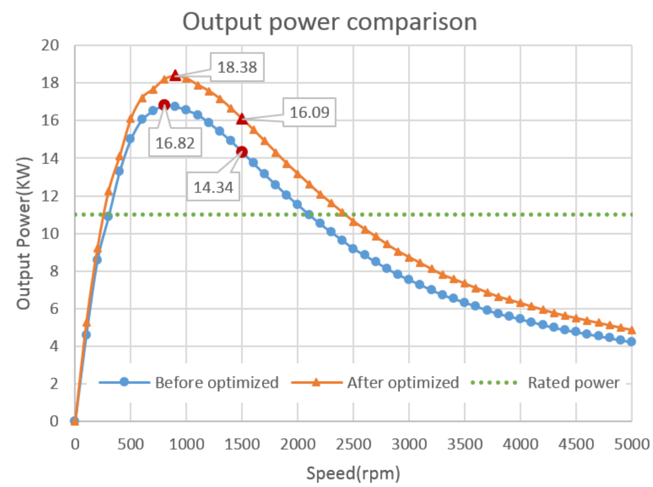


Figure 12 Output power comparison with various speed

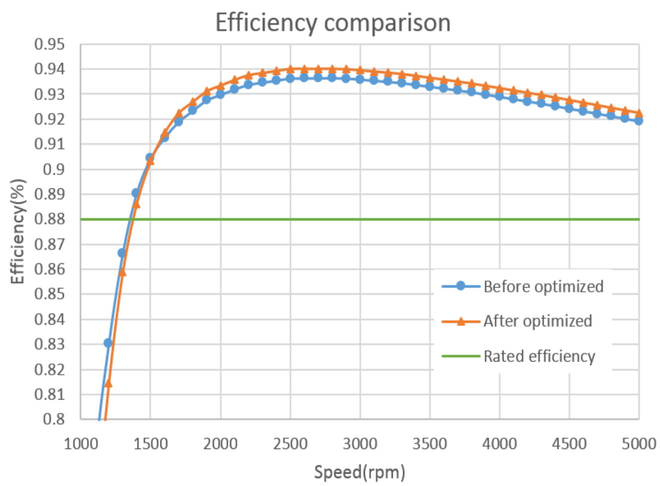


Figure 13 Efficiency comparison with various speed

V. CONCLUSION

This paper proposes a regulation model of ampere density and coil space factor to optimize the winding design of SRM. The purpose is to improve the performances through choosing the optimal number of winding turns, the cross-sectional area of conductor, which in turn leads to the optimal ampere density and coil space factor. It ultimately verifies that, after optimized, the ampere density is effectively decreased, which is beneficial for reducing the windings loss and increasing the efficiency. And the coil space factor is appropriately increased, which leads to the improvement of thermal performance of SRM on the other hand. Besides, not only the torque density, but also the power density and efficiency performances can be further improved. The simulation indicates that the average output power can be improved by 12.2% with the proposed optimization model and the rated efficiency increases from 92.25% to 93.08%.

The future work is torque ripple optimization and noise suppression through optimal design of SRM, and thermal performance is still remaining to be analyzed.

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